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## THE DEPTH OF THE EFFECTIVE PLANE IN X-RAY CRYSTAL PENETRATION

BY F. C. BLAKE,

DEPARTMENT OF PHYSICS, OHIO STATE UNIVERSITY

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In determining the value of 'h' by means of X-rays Blake and Duane<sup>1</sup> (p. 636) found out by experiment that the 'depth of the effective plane' was 0.203 mm. for the case of calcite, using X-rays of a wave-length 0.454 Å. An attempt is made in this note to explain this theoretically.

Call  $\mu$  the coefficient of true absorption and  $r$  the reflection coefficient. Suppose a parallel beam of X-rays strikes the crystal face at glancing angle  $\theta$ . Then if  $A_0$  is the amplitude of the primary beam the total effect at the ionization chamber is the sum of the various reflections from all those planes of atoms that are able for any reason to play a part at the ionization chamber. Call the last plane of atoms that is thus effective the  $m$ th plane. Figure 1 will render the situation clear. Let  $AB-IJ$  represent the parallel beam of X-rays as determined by the slit-width  $s$ . The reflected beam  $bdf$ , for instance, is the sum of the various reflections at  $b$ ,  $d$ , and  $f$ . Ray  $AB$  suffers partial reflection at  $A$  and arrives at  $a$  with amplitude  $A_0 l^{-\mu d \csc \theta}$  where it again suffers reflection. The reflected part has the amplitude  $rA_0 l^{-\mu d \csc \theta}$  at  $a$  and by the time it gets to  $c$  its amplitude has been reduced to  $rA_0 l^{-2\mu d \csc \theta}$ . Thus the total amplitude along any reflected ray  $bdf$  situated a distance  $x$  away from the first reflected ray  $AI$  is

$$rA_0 (1 + e^{-2\mu d \csc \theta} + e^{-4\mu d \csc \theta} + \dots + e^{-2n\mu d \csc \theta}), \text{ in which } n = \frac{x}{2d \cos \theta}.$$

This gives for the amplitude of the ray  $bf$ ,

$$rA_0 \frac{1 - e^{-2(n+1)\mu d \csc \theta}}{1 - e^{-2\mu d \csc \theta}}, \text{ reducing to } rA_0 \frac{1 - e^{-\frac{\mu x}{\sin \theta \cos \theta}}}{2\mu d \csc \theta},$$

very approximately.

Call  $D_1$  the mean depth of the ray  $bdf$ . Then

$$rA_0 \frac{1 - e^{-\frac{\mu x}{\sin \theta \cos \theta}}}{2\mu d \csc \theta} = nrA_0 e^{-2\mu D_1 \csc \theta}.$$

Solving for  $D_1$  we have

$$D_1 = \frac{\sin \theta}{2\mu} \log_e \frac{\mu x}{\sin \theta \cos \theta \left(1 - e^{-\frac{\mu x}{\sin \theta \cos \theta}}\right)} \quad (1)$$

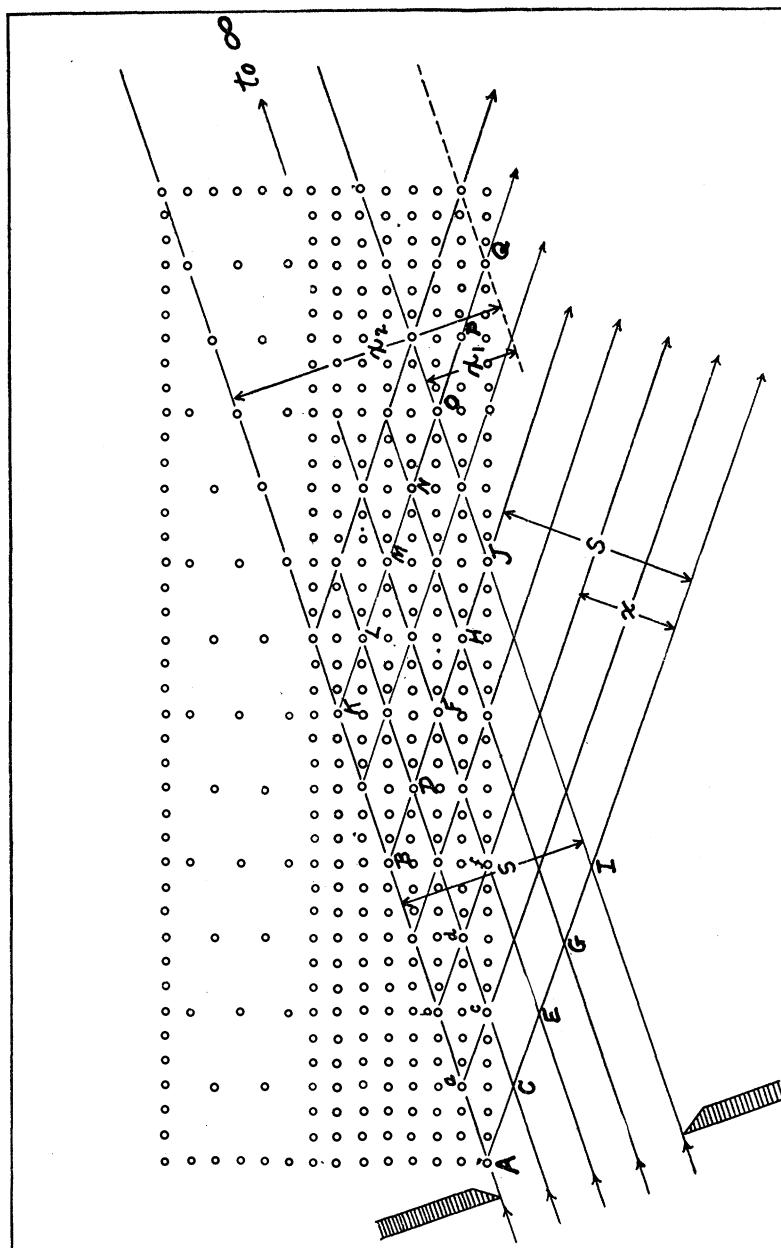


FIG. 1.

Now the 'effective plane' was defined by Blake and Duane<sup>1</sup> (p. 632) as that plane at which if reflection for all the rays occurred at *that* plane *only* the effect at the ionization chamber would be the same as actually does occur due to

the different reflections for all the planes of atoms playing any part. In accordance with this definition, if  $D$  is the depth of the 'effective plane' and if for the moment we limit ourselves to that portion of the reflected beam of width  $s$ , viz.,  $AI-BJ$ , it is clear, since for the separate rays we must add not amplitudes but intensities, that we must get  $D$  by integrating  $D_1$  throughout the region  $s$  and taking mean values. Thus if we were concerned only with the reflected beam of width  $s$  we could get  $D$  from (1) by replacing the parenthesis by

$$\left[ \frac{1}{s} \int_0^s \left( 1 - e^{-\frac{\mu x}{\sin \theta \cos \theta}} \right)^2 dx \right]^{\frac{1}{2}}.$$

But the reflected beam is not of width  $s$  only. Rather must we consider the effect of rays that penetrate the crystal to depths much greater than the plane through  $B$ . Consider for instance the ray  $AB$  penetrating to the plane through  $K$  say, where it is reflected along the direction  $KQ$  and emerges from the crystal at  $Q$ . At  $L, M, N, O$  the ray  $KQ$  is reinforced by reflections in phase with one another, but at  $P$  and  $Q$  there are no reinforcements since the initial beam is determined by the slit width  $s$ . Accordingly the amplitude of the ray  $KQ$  upon emergence is

$$rA_0 (e^{-2n\mu d \csc \theta} + e^{-2(n-1)\mu d \csc \theta} + \dots + e^{-2q\mu d \csc \theta}),$$

where  $q$  is the number of the plane through  $O$ . In other words the amplitude of the ray  $KQ$  upon emergence is

$$e^{-\frac{\mu x_1}{\sin \theta \cos \theta}} - e^{-\frac{\mu x_2}{\sin \theta \cos \theta}},$$

where  $x_1$  is the distance (measured parallel to the slit  $s$ ) between the incident ray through  $Q$  and that through  $O$ , and  $x_2$  is the distance between the ray through  $Q$  and that through  $K$ . Necessarily  $x_2 - x_1$  must equal  $s$ . Accordingly to get our mean penetration  $D$  for all the rays that penetrate into the crystal beyond the reflected ray  $BJ$  we must replace the parenthesis in (1) by

$$\left[ \frac{1}{x_2 - x_1} \int_s^\infty \left( e^{-\frac{\mu x_1}{\sin \theta \cos \theta}} - e^{-\frac{\mu x_2}{\sin \theta \cos \theta}} \right)^2 dx \right]^{\frac{1}{2}}.$$

Having found the mean penetration for all the rays between 0 and  $s$  and that for all the rays between  $s$  and  $\infty$  it is a simple matter to get the mean of these means, which should be, finally, the depth of the 'effective plane.'

Now in the above expressions the value of  $\mu$  is the amplitude coefficient of absorption while what is experimentally measured is the intensity coefficient of absorption. If we accordingly replace  $2\mu$  in the above expression by  $\mu$  we have as our expressions for the depth of the effective plane the following:

For the X-rays contained in the triangle  $ABJ$ ,

$$D' = \frac{\sin \theta}{\mu} \log_e \frac{\mu s}{2 \sin \theta \cos \theta \left[ \frac{1}{s} \int_0^s \left( 1 - e^{-\frac{\mu x}{2 \sin \theta \cos \theta}} \right)^2 dx \right]^{\frac{1}{2}}}; \quad (2)$$

and for the X-rays contained in the parallelogram  $BK \infty \infty OJB$ ,

$$\begin{aligned} D'' &= \frac{\sin \theta}{\mu} \log_e \frac{\mu s}{2 \sin \theta \cos \theta \left[ \frac{1}{s} \int_s^{\infty} \left( e^{-\frac{\mu x}{2 \sin \theta \cos \theta}} - e^{-\frac{\mu(x+s)}{2 \sin \theta \cos \theta}} \right)^2 dx \right]^{\frac{1}{2}}} \\ &= \frac{\sin \theta}{\mu} \log_e \frac{\mu s}{2 \sin \theta \cos \theta \left( 1 - e^{-\frac{\mu s}{2 \sin \theta \cos \theta}} \right) \left[ \frac{1}{s} \int_s^{\infty} e^{-\frac{\mu x}{\sin \theta \cos \theta}} dx \right]^{\frac{1}{2}}}. \end{aligned} \quad (3)$$

Now Blake and Duane, in determining the maximum positions of the ionization chamber corresponding to a given value of  $\theta$  had in one case made  $\theta$  equal to  $4^{\circ}18'$ , which corresponded to a wave-length  $\lambda$  equal to  $0.454 \text{ \AA}$ . Using Duane's formula for the absorption, viz.,  $\mu/\rho_{\text{Al}} = 14.9 \lambda^3$  we get  $\mu/\rho_{\text{Al}} = 1.394$ . Applying Bragg's formula for the atomic absorption coefficient,  $a$ , say, we have

$$a \equiv \mu \omega / \xi = k N^4,$$

where  $k$  is a constant and  $\omega$  the atomic weight. Thus we get

$$a_{\text{Al}} = 37.79 \text{ } A^{-1}, A_{\text{Ca}} = 211.7 \text{ } A^{-1}, a_{\text{C}} = 1.71 \text{ } A^{-1}, a_{\text{O}} = 16.26 \text{ } A^{-1},$$

where  $A$  is the number of molecules in a gram molecule, viz.  $6.062 \times 10^{23}$ . If now we assume the molecular absorption,  $m$  say, to be equal to the sum of the atomic absorptions, we have

$$m_{\text{CaCO}_3} = a_{\text{Ca}} + a_{\text{C}} + 3a_{\text{O}} = 229.67 \text{ } A^{-1},$$

whence  $\mu/\rho_{\text{CaCO}_3} = 229.67/100.07 = 2.295$ ; and if we take  $\rho$  for calcite to be 2.712,  $\mu$  comes but equal to 6.22.

Now W. L. Bragg has shown<sup>2</sup> that for calcite there is only half a molecule to each elementary cell. In other words, the value of  $\mu$  we require is  $6.22: \sqrt[3]{2} = 4.94$ .

Performing the integrations indicated in (2) and (3) we get

$$D' = \frac{\sin \theta}{\mu} \log_e \frac{\mu s}{2 \sin \theta \cos \theta \left[ \frac{1}{s} \left\{ \mu s + \sin \theta \cos \theta \left( 4e^{-\frac{\mu s}{2 \sin \theta \cos \theta}} - e^{-\frac{\mu s}{\sin \theta \cos \theta}} - 3 \right) \right\} \right]^{\frac{1}{2}}}; \quad (4)$$

and

$$D'' = \frac{\sin \theta}{\mu} \log_e \left[ \left( \frac{\mu s}{\sin \theta \cos \theta} \right)^{3/2} \frac{1}{\left( 1 - e^{-\frac{\mu s}{2 \sin \theta \cos \theta}} \right) e^{-\frac{\mu s}{2 \sin \theta \cos \theta}}} \right].$$

Now in the work of Blake and Duane  $s$  was 0.040 cm. Taking  $\mu = 4.94$  and  $\theta = 4^\circ 18'$ ,  $D'$  comes out 0.150 mm.; and  $D''$ , 0.363 mm.

If we plot the relative intensity, viz.

$$\left(1 - e^{-\frac{-\mu x}{2 \sin \theta \cos \theta}}\right)^2,$$

against  $x$ , we get Curve I of figure 2 for values of  $x$  less than 0.04 cm. For values of  $x$  greater than 0.04 cm. the relative intensity is

$$\left(e^{-\frac{\mu x}{2 \sin \theta \cos \theta}} - e^{-\frac{\mu(x+s)}{\sin \theta \cos \theta}}\right)^2,$$

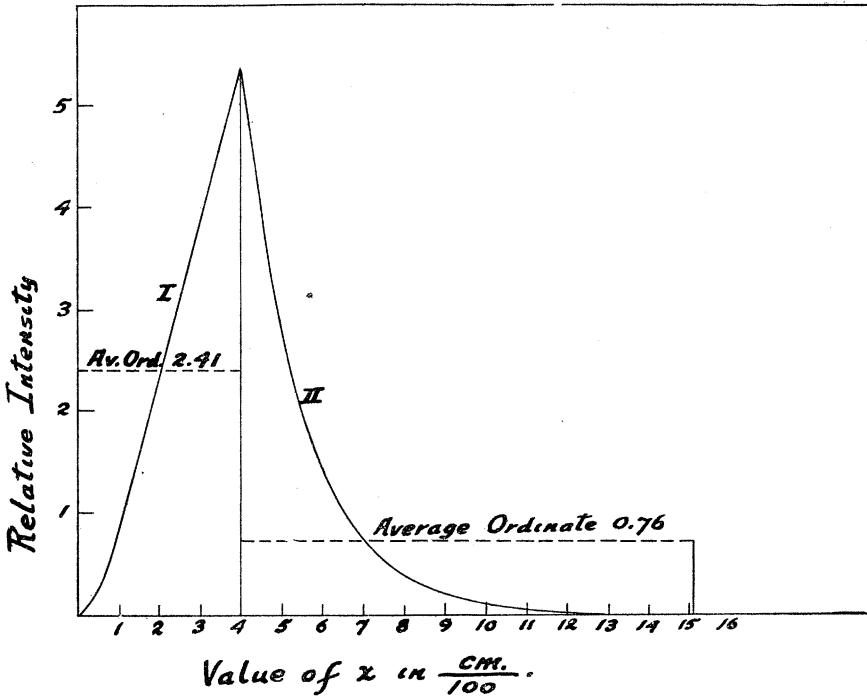


FIG. 2.

and plotting this against  $x$ , we get Curve II of figure 2. Measured carefully with a planimeter the average ordinate for I is 2.41 and for II it is 0.76. Finally therefore we have as the depth of the effective plane

$$D = \frac{2.41 D' + 0.76 D''}{3.17} = 0.201 \text{ mm.}$$

Blake and Duane found  $D$  experimentally to be 0.203 mm. Thus the agreement is all that can be expected.

In figure 1, for simplicity sake the writer has assumed the space lattice for calcite to be cubical; as a matter of fact it is rhombohedral. Blake and Duane

pointed out<sup>3</sup> that, when crystal penetration is properly corrected for, the curve of atomic number versus square root of frequency is *almost* a straight line, their own values falling more nearly on a straight line than either those of Moseley or de Broglie. As a matter of fact it is manifest that the effective reflected ray for a crystal like calcite is along such a direction as to make the measured angle of reflection proportionately too large for the higher frequencies. In other words as the atomic number increases the measured frequency is proportionately greater than it should be. This possibly accounts for the fact that the curve in figure 3 of Blake and Duane's paper is convex downward toward the axis of atomic numbers. It would seem that a test of this reasoning could be made by repeating the work of Blake and Duane using a crystal with a cubic lattice but one whose faces are as good as those of calcite (rock salt is notoriously bad), care being used of course to correct for crystal penetration, or to eliminate the effect of such penetration by using a very narrow incident beam and having the ionization chamber slit very wide (the arrangement shown in figure 1, first used by Duane and Hunt<sup>4</sup>).

<sup>1</sup> *Physic. Rev. Ithaca*, **10**, 1917, (624-637).

<sup>2</sup> *Proc. Roy. Soc., (A)*, **89**, 1914, (468).

<sup>3</sup> *Physic. Rev. Ithaca*, **10**, 1917, (703).

<sup>4</sup> *Ibid.*, August, 1915, p. 166.

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THE MYODOME AND TRIGEMINO-FACIALIS CHAMBER OF  
FISHES AND THE CORRESPONDING CAVITIES IN  
HIGHER VERTEBRATES

BY EDWARD PHELPS ALLIS, JR.

PALAIS CARNOLÉS, MENTON, FRANCE

Communicated by E. L. Mark, June 20, 1918

A functional myodome is found only in fishes, and even among them it is limited, in the specimens I have examined, to *Amia* and the non-siluroid Teleostei.

The myodome is always separated from the *cavum cerebrale* of the skull by either membrane (dura mater), cartilage or bone, and the separating wall is in part spinal and in part prespinal in position. A depression in the prespinal portion lodges the hypophysis, or both the hypophysis and *saccus vasculosus*, and this part of the wall never undergoes either chondrification or ossification, a more or less developed pituitary sac always projecting into the myodome.

The myodome is found in its most complete form in the Teleostei, and there consists of dorsal and ventral compartments, which are usually separated from each other by membrane only, but that membrane, the horizontal myodomic membrane, is capable of either chondrification or ossification. The dorsal